PERGAMON

International Journal of Multiphase Flow 26 (2000) 169–190

International Journal of Multiphase Flow

www.elsevier.com/locate/ijmulflow

The roles of interfacial stability and particle dynamics in multiphase flows: a personal viewpoint ☆

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Received 18 October 1998; received in revised form 28 December 1998

Abstract

Significant progress in understanding multiphase flows has been made by relating macroscopic properties to small scale behavior. This is illustrated by considering the impact of interfacial stability on the prediction of flow regimes and the use of an understanding of the influence of fluid turbulence on entrained particles to describe rates of deposition.

1. Introduction

The Lyon Conference on Multiphase Flow has a kinship to previous international meetings, which have shaped my career. A symposium held at Exeter (P.M.C. Lacey) in 1965 brought together 160 people with a wide range of interests. Discussions at the 42 presentations indicated, to me, that something special was happening and that future directions of work in multiphase flow were being defined. This thrust was continued by conferences at Waterloo, Canada, in 1965 (E. Rhodes, D.S. Scott) and at Haifa in 1971 (G. Hetsroni). Intellectual activity in ensuing years is exemplified by more focussed conferences on Annular and Dispersed Flows held in Pisa in 1984 (S. Zanella, P. Andreussi, T.J. Hanratty) and in Oxford in 1987 (G. Hewitt, P. Whalley, B. Azzopardi), the Symposium on Measuring Techniques in Nancy in 1983 (J.M. Delhaye) and the very recent conference on Gas Transfer at Heidelberg

☆ Plenary lecture presented at Third International Conference on Multiphase Flow, ICMF'98, Lyon, France, June 8–12, 1998.

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0301-9322/00/$ - see front matter
PII: S0301-9322(99)00004-X

(Jähne, 1995). However, the 350 papers presented at the Second International Conference on Multiphase Flow at Kyoto in 1995 (A. Serizawa, Y. Tsuji) manifested a new level of activity.

A fair question is what has happened between 1965 and 1995. My own assessment is that major successes have come about, mainly, through efforts that relate macroscopic properties of multiphase systems to small scale behavior. An outcome of this approach is the possible emergence of a new field. This is evidenced in many ways, of which the establishment of the International Journal of Multiphase Flow (Gad Hetsroni, 1973) and the Japan Society of Multiphase Flow (A. Akagawa, T. Fukano, 1987) are examples. The following excerpt from comments made by R.T. Lahey at the inauguration of the Japan Society would indicate that my observations are not original: "I believe that this new field will become as widely accepted in the future as other emerging fields...".

What is needed to bring this about are texts which reflect advances made since the publication of Graham Wallis' textbook on One-Dimensional Two-Phase Flow in 1969. The book on Multiphase Flow with Droplets and Particles by C. Crowe, M. Sommerfeld and Y. Tsuji that was published in 1998 appears, to me, to have the correct approach. The intellectual backbone of work in multiphase flows is basic knowledge on waves, particle (solid, liquid or gaseous) dynamics, interfacial phenomena (dropsize, coalescence, etc.), computational methods (two fluid models, Stokesian dynamics, etc.), heat/mass transfer with possible phase change and instrumentation. Successful applications of this knowledge are a measure of the advancement of the 'field.' However, such applications often are not direct, so a challenge in multiphase research is the formulation of tools that can be used by the practitioner. Future progress requires advances both in fundamental knowledge and in the artful use of this knowledge.

This paper illustrates this progress by considering flow regimes in horizontal and near horizontal flows and the rate of deposition of particles. It also includes some surprising new results on the roles of interfacial stability and particle dynamics, which appear in Ph.D. theses by Bennett Woods and Ilias Iliopoulos (1998).

2. Flow regimes in horizontal flows

2.1. Inviscid linear stability of a stratified flow

Gas and liquid flowing in a horizontal pipe can assume different configurations which are characterized as stratified, annular, slug, plug and homogeneous flows. The prediction of the transition from one regime to another is of central importance. Early work characterized the flow regimes by a mapping technique. One of the first contributions in this direction was made by Baker (1954). A later one by Mandhane et al. (1974) used the superficial gas and liquid velocities as the ordinates for the map. The difficulty with this approach is that the mappings vary with fluid properties and pipe diameter in ways that are not easily understood. Attempts to develop a physical basis for this map were initiated by Kordyban and Ranov (1970) and Wallis and Dobson (1973). In particular, inviscid analyses for the stability of a stratified flow were used to define the transition to a slug flow.

An inviscid linear analysis of wave motion on a liquid layer with height h_L and velocity u_L gives the following relation between the complex wave velocity, c, and the wave number, k:

$$k\rho_G(u_G - c)^2 \coth(kh_G) = g\cos\theta(\rho_L - \rho_G) + \sigma k^2 - k\rho_L(u_L - c)^2 \coth(kh_L), \quad (1)$$

where h_G is the height of the gas space, u_L, the liquid velocity, ρ_L, the liquid density, ρ_G, the gas density, σ, the surface tension, g the acceleration due to gravity and $\theta = 0°$ for a horizontal flow. At neutral stability, $c_I = 0$ and (1) gives the critical Kelvin–Helmholtz gas velocity. The first and second terms on the right side of (1) give the stabilizing effects of gravity and surface tension; the third is the destabilizing effect of liquid inertia. The critical gas velocity and the wavelength calculated for air and water at atmospheric pressure are $u_G \cong 6.8$ m/s, $\lambda \cong 2$ cm.

The argument was made that slugs evolve from long wavelength waves so surface tension effects can be neglected and the assumption of $kh_L \ll 1$ and $kh_G \ll 1$ can be made. Eq. (1), therefore, simplifies to the following relations

$$c_R \cong u_L \quad (2)$$

$$\rho_G(u_G - u_L)^2 = h_G\rho_L g\cos\theta \quad (3)$$

This equation predicts gas velocities which are larger than the critical Kelvin–Helmholtz condition; yet, slugging is observed for smaller u_G than 6.8 m/s for air–water flows. Wallis and Dobson showed that the gas velocity required for the initiation of slugs is approximately 1/2 of that predicted by Eq. (3). This has been explained by arguing that a nonlinear long wavelength analysis needs to be used and that transition occurs when the velocity of the gas at the crest is roughly equal to the critical value predicted by (1). The difficulty with this approach is that assumptions need to be made regarding the wavelength of the waves and/or their height.

2.2. Taitel–Dukler analysis

A significant advancement in developing an overall scheme for the prediction of flow regimes was made by Taitel and Dukler (1976). The main contribution in this paper is that theoretical equations based on a phenomenological picture of the transitions were developed. Different maps for different fluid pairs could then be constructed. A centerpiece of their approach is the inviscid analysis of the stability of a stratified flow. Eq. (3) was adapted to be used in a circular pipe and the factor $[1 - (h_L/D)]$ was introduced on the right side to account for the limitations of linear theory. This predicted an initiation of slugging at lower u_G than that is suggested in Eq. (1).

Taitel and Dukler assumed that this instability can lead to a transition either to an annular flow, if $h_L/D < 0.5$, or to an intermittent pattern, which includes both plug and slug flows. An important prediction is that transition to intermittent flow occurs at higher gas rates as the pipe diameter increases. The transition to a homogeneous pattern was suggested to occur when the liquid flow is large enough that turbulence can prevent bubbles from rising to the top of the pipe.

2.3. Advances in physical understanding

The success of the Taitel–Dukler analysis for flow regimes is evidenced by the fact that it is widely used today, with some modifications. However, physical understanding of the transitions have improved considerably over the past 20 years, so there is an opportunity to develop a new scheme for predicting flow regime maps. An account of these advances will now be given.

A long wavelength linear stability analysis that includes the influences of interfacial and wall shear stresses was examined by Lin and Hanratty (1986). This differs from the inviscid analysis in that $c_R \neq u_L$ so that destabilizing effects of the term, in (1), representing liquid inertia is not zero. A consequence is that lower gas velocities are predicted. These were found to agree with observations made for air and water flowing at atmospheric pressure in horizontal pipes with diameters of 2.54 and 9.5 cm. This supported the notion that a linear stability analysis of a stratified flow can be used to predict the initiation of slug flow under these conditions.

However, studies by Fan et al. (1993a) of the initiation of slugs, for air/water flows in a 9.5 cm pipe at superficial gas velocities less than 3 m/s show a different mechanism for the formation of slugs than would be anticipated from the linear analysis. At the transition, small amplitude waves with a very long length appeared at the interface. However, slugs did not evolve from these waves but from waves, with lengths of 8–10 cm, that are generated by gas phase pressure variations in phase with the wave slope (Cohen and Hanratty, 1965). These waves grow in height and double in wavelength by a nonlinear resonance mechanism. Depending on the height of the liquid, the growth can lead to a breaking wave or to a wave that fills the whole pipe cross section. The reconciliation of these results with the good predictions based on a linear stability analysis has not been accomplished–although recent observations (Woods, 1998) of the initiation of slugs in inclined pipes could offer an explanation (see Section 5).

Perhaps, the most important advance in our understanding of flow regimes is the recognition that a consideration of the stability of a slug, is at least, as important as a consideration of the stability of a stratified flow. Ruder et al. (1989) developed necessary conditions for the existence of slugs. The front of a slug was defined as a sudden expansion (or a hydraulic jump) seen by an observer moving with the slug; Froude number criteria could then be developed to predict whether an intermittent flow involves a slug or a plug (Ruder and Hanratty, 1990). By considering conservation of mass in a system moving with the slug a criterion for stability was developed by picturing the back of the slug to be a moving bubble. The consequence of this analysis is that the liquid layer behind the tail of the slug will have a height, h_0/D, which depends on the shedding rate of the slug. If the height of the liquid layer in front of the slug is greater than h_0 the slug will grow; if it is less, the slug will decay. Clearly, slugs could not be generated on a stratified flow with a height less than h_0/D, so this becomes a necessary (but not a sufficient) condition for the existence of a slug.

Ruder et al. (1989) used a Benjamin bubble to represent the back of a slug. Values of h_0/D, calculated for air and water at atmospheric pressure, were found to be less than the critical height, h_s/D, predicted by a consideration of the stability of a stratified flow. Slugs formed through such an instability would, therefore, satisfy the necessary condition. Since the condition for slug stability is derived from kinematic considerations it is insensitive to changes in gas density. However, the stability of a stratified flow depends on the dynamics so one needs to use $\rho_G^{1/2}\bar{u}_G$ as a criterion. As a consequence, the u_G required for instability decreases with increasing ρ_G. As pointed out by Ruder et al., at large enough ρ_G a situation is reached for which $h_s < h_0$ so the long wavelength stability analysis cannot predict the initiation of slugs.

There is experimental evidence for the failure of the Taitel–Dukler analysis in large pressure applications. However, to my knowledge a detailed examination of whether the concept of slug stability can be used to predict transitions for conditions where ρ_G is large is not available.

As observed by Taitel and Dukler, a consideration of the stability of a stratified flow fails to predict the initiation of slugs at large gas velocities, where slugs are observed to originate from large amplitude irregular waves (Lin and Hanratty, 1987). These waves coalesce and break up at small h_L. However, a critical height h_{LC} could be observed at which coalescence lead to the formation of slugs. The work of Ruder et al. offered the possibility that transition could be predicted by considering the stability of a slug. However, calculations based on the model of Ruder et al. produced values of h_{LC} which were not correct. Studies of bubble motion in inclined tubes (Bendiksen, 1984) and the shedding rate (Woods and Hanratty, 1996), however, reveal that the Benjamin bubble is a good model for a slug tail only in the limit of small gas velocities. If more realistic models are used, the condition of slug stability is found to predict h_{LC} for the flow of air and water and for the flow of air and a 100 cp liquid (Woods and Hanratty, 1996). These studies of slug stability, thus, agree with the experimental observation that h_L/D must be greater than 0.2–0.3 for slugs to form at large u_G and provide a physical basis for the suggestion, by Taitel and Dukler, that there is a critical value of h_L/D below which slugs cannot form.

The success of the viscous analysis of the stability of a stratified flow motivated an experiment in which the influence of liquid viscosity was explored (Andritsos et al., 1989). For a fixed h_L/D and u_G the liquid velocity decreases with increasing viscosity. As a consequence, the destabilizing influence of liquid inertia becomes less. For large enough viscosities the long wavelength viscous analysis yields the inviscid result, Eq. (3)! Experiments show that viscous effects were predicted for liquid viscosities in the range of 1–10 cp. However, for viscosities greater than 20 cp, the viscous long wavelength analysis (as well as the Taitel–Dukler analysis) predicts critical gas velocities which are too large. Also, no effect of pipe diameter was observed for liquid viscosities greater than 20 cp, in contradiction to the long wavelength analysis.

The explanation of these results comes from the recognition that the flow is dominated by a Kelvin–Helmholtz instability when the gas velocity reaches the critical value predicted by (1). At this condition the interface is covered with capillary-gravity waves. For $h_L > h_0$ these evolve into slugs by a process which is not completely understood. For $h_L < h_0$ they evolve into large amplitude irregular waves which greatly increase interfacial drag (Andritsos and Hanratty, 1987a, 1987b).

Finally, some comments should be made regarding the change from a stratified flow to an annular flow. Lin and Hanratty (1987) made detailed observations about this transition for air and water flowing in horizontal 2.54 and 9.53 cm pipelines. The flow regime map of Taitel and Dukler uses the notion that the wetting of the top wall of the pipe is initiated by large amplitude waves which ring around the pipe circumference, as suggested by Butterworth and Pulling (1972), Govier and Aziz (1972) and Taitel and Dukler (1976). Hoogendoorn (1959), however, concluded that annular flow is caused by an entrainment-deposition mechanism. Measurements in a 9.53 cm pipe at all liquid rates and in a 2.54 cm pipe at moderate liquid rates agree with the Hoogendoorn mechanism. For high liquid rates in the 2.54 cm pipe, waves that ring the whole circumference help in wetting the top of the pipe. These experiments suggest that the transition to annular flow is best defined as the initiation of a condition at which atomization and deposition can keep the top wall wetted. This transition should not be defined by using a long wavelength instability of a stratified flow.

Studies by Andritsos and Hanratty (1987a) with 2.54 and 9.53 cm pipes and with liquids whose viscosity varied from 1 to 80 cp indicate that the transition occurs at a gas velocity which is about twice that needed to initiate atomization.

3. Particle deposition

3.1. Entrainment in annular flows

In the annular regime, observed for gas–liquid flows, part of the liquid moves along the wall as a film with a mass flow rate of W_{LF} and part as drops with a rate W_{LE}. The fraction of the liquid entrained as drops is then given as $E = W_{LE}/W_L$, where $W_L = W_{LE} + W_{LF}$. This regime occurs in many applications—but, unfortunately, is poorly understood. One of the critical issues is the prediction of E. Straightforward correlations of measurements of entrainment have not been successful. This has lead to an approach which describes W_{LE} as resulting from a balance between the rate of atomization of the liquid film, R_A, and the rate of deposition of drops, R_D. Early activity along these lines is amply demonstrated in papers presented at the Exeter and Waterloo Conferences and is described in the book on Annular Flow by Hewitt and Hall-Taylor (1970).

The rate of deposition is usually related to the bulk concentration of the drops, C_B, through a deposition parameter, k_D:

$$R_D = k_D C_B = k_D \frac{W_{LE}}{Q_G S} \tag{4}$$

where Q_G is the volumetric flow of the gas and S is ratio of the drop velocity and the gas velocity. Under fully developed conditions, $R_D = R_A$, so

$$W_{LE} = \frac{R_A Q_G S}{k_D} \tag{5}$$

The critical problem in implementing this approach is the development of physical models for k_D and for R_A. This section gives an account of progress that has been made in understanding deposition.

3.2. Models for particle turbulence

The study of Friedlander and Johnstone (1957) on the deposition of aerosols by turbulence is characteristic of early theoretical work, which is reviewed by McCoy and Hanratty (1977). The particles assume a turbulent motion by responding to the fluid velocity fluctuations. The ability of spherical particles of diameter d_p to respond to the turbulence is characterized by an inertial time constant defined as

$$\tau_p = \frac{4 d_p \rho_p}{3 C_D \rho_F}, \tag{6}$$

where C_D is the drag coefficient. Small τ_p means that the particle inertia is small enough that it follows the fluid turbulence closely. Aerosol particles are usually characterized by $\tau_p^+ < 20$,, where the time constant is made dimensionless with the friction velocity and the kinematic viscosity of the fluid. For $\tau_p^+ <$ ca 0.1, deposition occurs by Brownian motion. For $0.1 < \tau_p^+ < 10$, the particles follow the fluid velocity fluctuations in the outer flow. However, they become disengaged from the turbulence close to the wall, where normal velocity fluctuations are very small, and deposit on the wall by what is called a 'free-flight'. For $\tau_p^+ >$ ca 20 particles start their free-flight beyond $y^+ = 30$ so that, unlike aerosol particles, their deposition rate is not strongly dependent on the details of the turbulence nonhomogeneities that exist in the viscous wall layer. Droplets in annular flow are usually characterized by large τ_p^+; the root-mean square of their velocity fluctuations and the scale of their motion can be quite different from the fluid turbulence which is causing their haphazard trajectories. The understanding of droplet deposition, therefore, requires an understanding of how the particles respond to the fluid turbulence.

Friedlander (1957) considered the behavior of spherical particles in a homogeneous isotropic turbulence by solving the equation of motion,

$$\frac{d\vec{v}}{dt} = \beta(\vec{u} - \vec{v}), \tag{7}$$

where β is the inverse of the inertial time constant of the particles and $\vec{u}$ is the randomly varying fluid velocity. He showed that dispersion of particles from a point source is characterized by a turbulent diffusivity given as

$$\varepsilon_p = \overline{u^2} \int_0^\infty R(\theta)\, d\theta \tag{8}$$

where $R(\theta)$ is different from the usual Eulerian or Lagrangian correlation coefficients. It represents the correlation of the fluctuating velocities of the fluid seen by the particles, whose paths are unknown. If $R(\theta)$ is represented by an exponential function, $\exp(-\alpha\theta)$, the turbulent diffusion coefficient of the particles is given as

$$\varepsilon_p = \frac{\overline{u^2}}{\alpha} = \frac{\varepsilon_f}{\alpha \tau_{LF}}, \tag{9}$$

where τ_{LF} is the Lagrangian time scale of the fluid turbulence. The root-mean square of the particle velocity fluctuations is related to the fluid velocity fluctuations by

$$\overline{v^2} = \overline{u^2}\left(\frac{\beta}{\alpha + \beta}\right) = \overline{u^2}\left(\frac{\beta \tau_{LF}}{\alpha \tau_{LF} + \beta \tau_{LF}}\right) \tag{10}$$

The application of these results was handicapped because of uncertainties in relating $R(\theta)$ to Eulerian or Lagrangian properties of the fluid turbulence. A solution to this problem came with the analyses of Reeks (1977) and of Pismen and Nir (1978). Reeks characterized the fluid turbulence by the mean-square of the fluid velocity fluctuations and a characteristic wave number, k_0, and developed the following relations:

$$\frac{\overline{v^2}}{\overline{u^2}} = f_1\left(\frac{\beta}{k_0\left(\overline{u^2}\right)^{1/2}}, \frac{u_T}{\left(\overline{u^2}\right)^{1/2}}\right) \tag{11}$$

$$\frac{\varepsilon_p}{\bar{\varepsilon}_f} = f_2\left(\frac{\beta}{k_0\left(\overline{u^2}\right)^{1/2}}, \frac{u_T}{\left(\overline{u^2}\right)^{1/2}}\right) \tag{12}$$

where u_T is the relative velocity between the particles and the fluid, caused by the gravity. For $u_T = 0$, $\overline{v^2}/\overline{u^2}$ is found to decrease with decreasing $\beta/k_0(\overline{u^2})^{1/2}$ and $\varepsilon_p/\varepsilon_f$ is found to vary only by about 30% for the whole range of $\beta\tau_{LF}$. That is, $\alpha = \tau_{LF}^{-1}$ for $\beta\tau_{LF}\rightarrow\infty$ and $\alpha\tau_{LF} \cong 1.3$ for $\beta\tau_{LF}\rightarrow 0$. Thus, for $u_T = 0$, increases of the inertial time scale of the particles is accompanied by an increase in the characteristic length scale of the particle turbulence, a decrease in $(\overline{v^2})^{1/2}$ and a relatively constant value of the product.

3.3. Experimental studies of particle turbulence

These results prompted investigations in our laboratory of the behavior of spherical drops injected from a small diameter tube into a gas flowing in a pipe (Lee et al., 1989a, 1989b). These experiments were carried out over a long enough length of pipe that droplet deposition occurred. This could be measured by putting dye into the droplets. The droplets hitting the wall evaporated. At the end of a run the nine sections of the pipe were disassembled and washed to determine the amount of ink that deposited. Experiments were also carried out in which glass and stainless steel spheres were injected into water at the center of a pipe (Young and Hanratty, 1991a). Far enough downstream the stainless steel spheres reached the wall and became trapped in the viscous sublayer (Young and Hanratty, 1991b). Axial viewing optical techniques allowed measurements of the root-mean square of the fluctuations in the r- and θ-components of the velocity and in the r- and θ-components of the acceleration. The flows were fully developed so the mean fluid velocity in the radial direction was zero. However, mean values of the particle velocity and acceleration in the radial direction were measured. Turbulent diffusivities could be determined from the first measurement and concentration profiles, since

$$\varepsilon_P = \bar{V}_R\bar{C}/\left(-\frac{d\bar{C}}{dr}\right) \tag{13}$$

The measurements of the mean acceleration provided a direct demonstration of the turbophoretic phenomenon whereby particles migrate from regions of high turbulence to regions of low turbulence. These experiments had an important impact on our understanding of the physics of particle deposition. Some of the results are summarized here.

The ability of the particles to follow the turbulent fluctuations can be characterized by the parameter $\beta\tau_{LF}$, where τ_{LF} is the Lagrangian time scale of the fluid turbulence. Measurements of $\overline{v_r^2}$ for small u_T agree with (10) if $\alpha\tau_{LF} \cong 0.7$ measurements of $\overline{v_\theta^2}$ agree if $\alpha\tau_{LF} \cong 0.8$. Over the range of $\beta\tau_{LF}$ characterizing annular flow, $(\overline{v_r^2})^{1/2}/(\overline{u_r^2})^{1/2}$ is of the order of 0.1, so the particles are sluggishly following the fluid.

The experiments agree with the predictions of Friedlander and of Reeks in that $\varepsilon_p \cong \varepsilon_F$ if $u_T = 0$. However, ε_p can be much smaller than ε_F if $u_T \neq 0$. This is particularly evident in the experiments with solid spheres in a water flow. These are characterized by large $\beta\tau_{LF}$ for which $\alpha\tau_{LF} = 1$ for $u_T = 0$. A comparison of measurements of ε_p for $u_T/u^* = 0.13$ and for $u_T/u^* = 1.77$ show a decrease of $\alpha\tau_{LF}$ from 1 to 0.6, in agreement with the 'crossing of trajectories' concept of Yudine (1959) and the calculations of Reeks (1977). The influence of the crossing of trajectories on the particle motion is manifested by an increase in the root-mean-square of the fluctuations in the particle acceleration (Young and Hanratty, 1991a). Consistent with (10), a decrease in $\alpha\tau_{LF}$ by 40 per cent was observed to have little effect on $(\overline{v_r^2})/(\overline{u_r^2})$, since $\beta\tau_{LF}$ was very large.

An important aspect of these experiments is that the analysis of Reeks provides a good approximation for the turbulence properties of particles and drops in the central region of a pipe if his time scale $(k_0u_0)^{-1}$ is set equal to 0.9–1.0τ_{LF} (Lee et al., 1989a, 1989b; Hay et al., 1996).

3.4. Deposition in vertical annular flow

For annular flows, the very complex properties of the turbulent field in the viscous wall region need not be taken into account. Lee et al. (1989a), therefore, explored the use of a diffusion model to describe the deposition of drops, originating from a small source in the center of a pipe by assuming that the flow is uniform. This simplification receives support from the experiments of Lee et al. (1989b) which show small variations of $\overline{v_r^2}$ and $\overline{v_\theta^2}$ over the pipe cross section for the range of $\beta\tau_{LF}$ characteristic of annular flows. A critical issue in formulating this model is the specification of the boundary condition at the wall. The wall was considered to be a perfect sink. For situations involving molecular diffusion this would require that the concentration at the wall be zero. However, because the length scale characterizing the droplet turbulence, L, is so large a finite concentration exists at the wall.

This can be seen by using the boundary condition,

$$-\varepsilon_p \frac{\partial C}{\partial r}\Big|_{r=R} = VC(R) \tag{14}$$

where V is the velocity with which droplets are carried to the wall. If ε_p is represented as the product of V and a characteristic length L then (14) may be rewritten as

$$VL\left(\frac{\partial C}{\partial r}\right)_{r=R} \cong VC(R) \tag{15}$$

If L is very small (as is the case for molecular diffusion) and $(\frac{\partial C}{\partial r})_R$ is finite, then $C(R) = 0$. However if L is the order of the length scale characterizing $\partial C/\partial r$, then $C(R)$ is a finite number. If we set $V = A(\overline{v_r^2})^{1/2}$ in (15), then the boundary condition becomes

$$-\varepsilon_p \frac{\partial C}{\partial r}\Big|_{r=R} = A\left(\overline{v_r^2}\right)^{1/2} C(R) \tag{16}$$

Now, if v_r can be represented by a distribution function with a mean of zero,

$$V = \int_0^\infty v_r P(v_r)\, \mathrm{d}v_r \tag{17}$$

If the distribution function is Gaussian, $A = (2\pi)^{-1}$.

As shown by Lee et al. (1989a), (1989b) the solution of the diffusion equation with b.c. (16) gives a relation

$$\frac{1}{k_\mathrm{D}} = \frac{1}{k_\mathrm{D1}} + \frac{1}{k_\mathrm{D2}}, \tag{18}$$

which is the sum of resistances representing diffusion and free-flight to the wall. Good agreement with measured deposition rates was observed for 50 μm drops, for which $(\overline{v_\mathrm{r}^2})^{1/2}/(\overline{u_\mathrm{r}^2})^{1/2} \cong 0.8$ and $u_\mathrm{T}/(\overline{v_\mathrm{r}^2})^{1/2} \cong 0.18$, if $\varepsilon_\mathrm{p}/\varepsilon_\mathrm{p} \cong 1.03$. Experiments with 90 μm drops, for which $(\overline{v_\mathrm{r}^2})^{1/2}/(\overline{u_\mathrm{r}^2})^{1/2} \cong 0.5$ and $u_\mathrm{T}/(\overline{v_\mathrm{r}^2})^{1/2} \cong 0.39$, gave $\varepsilon_\mathrm{p}/\varepsilon_\mathrm{p} \cong 0.89$. Experiments with 150 μm drops, for which $(\overline{v_\mathrm{r}^2})^{1/2}/(\overline{u_\mathrm{r}^2})^{1/2} \cong 0.33$ and $u_\mathrm{T}/(\overline{v_\mathrm{r}^2})^{1/2} \cong 1.0$, gave $\varepsilon_\mathrm{p}/\varepsilon_\mathrm{p} \cong 0.48$. An effect of $u_\mathrm{T}/(\overline{v_\mathrm{r}^2})^{1/2}$ is clearly shown.

In all of the above, a single dropsize was considered. The theory can easily be extended to include a distribution by adding the weighted contributions of each dropsize to the deposition.

Again, to simplify the analysis, annular flows will be considered to be composed of a single dropsize. Usually the Sauter mean diameter is employed since it gives a stronger weighting to large diameter drops which are carrying most of the mass. Annular flows are clearly more complicated than the experiments of Lee et al. (1989a), in that they should be described as resulting from a series of wall sources (Binder and Hanratty, 1991, 1992). However, if only a fully developed flow is considered, one can avoid the issues associated with modelling the diffusion process, since $(\overline{v_\mathrm{r}^2})$ is very small and the controlling resistance is the free-flight to the wall. This is evidenced by the measured flat concentration profiles (Hay et al., 1996; Gill et al., 1964). The deposition parameter may be taken as

$$k_\mathrm{D} = k_\mathrm{D2} = (2\pi)^{-1}\left(\overline{v_\mathrm{r}^2}\right)^{1/2} \tag{19}$$

Thus, the prediction of k_D involves the evaluation of $\overline{v_\mathrm{r}^2}$. For dilute suspensions, $\overline{v_\mathrm{r}^2}$ can be calculated with (10) and estimates $\alpha\tau_\mathrm{LF}$ can be obtained from the work of Reeks.

However, the use of (10) assumes that the drops have been in the field long enough that a stationary state is reached. This might not, always, be the case. In general, this requires that the length characterizing the particle turbulence is not large compared to the pipe radius (Binder and Hanratty, 1991). In the case of annular flow, drops enter the field with a velocity in the radial direction v_0. The attainment of a stationary state requires that the droplet turbulence is independent of v_0 and that k_D is not affected by v_0. A number of researchers (James et al., 1980; Andreussi and Azzopardi, 1983; Russell and Lamb, 1965; Whalley et al., 1979) have argued that under some circumstances k_D is completely dependent on v_0 and that the droplets are undergoing unidirectional motion from one wall to another.

If one makes the simplifying assumptions that drops move a distance $2R$ in traveling from one wall to another and that resistance to their motion is the same as would be experienced in quiescent flow, a criterion can be established for v_0 to be affecting deposition.

$$\frac{v_0}{2R\beta} > 1 \tag{20}$$

James et al. (1980) and Andreussi and Azzopardi (1983) have suggested that $v_0 \cong cv^*$, where c is a constant of order one. If $v^*/2R$ is taken as $0.046/\tau_{LF}$ (Binder and Hanratty, 1991), then (20) gives

$$\beta\tau_{LF} > (0.046c) \tag{21}$$

For conditions under which this mechanism is completely controlling, the velocity with which droplets hit the wall can be estimated as

$$v_{0w} = v_0\left[1 - \left(\frac{2R\beta}{v_0}\right)\right] \tag{22}$$

Again, if v_0 is assumed equal to cv^* the argument $v_0/2R\beta$ can be approximated by $0.046c/\beta\tau_{LF}$. Eq. (21) must be considered tentative since verifying experiments are not available.

The application of dilute theory requires methods to predict the droplet size distribution and the slip ratio S. It also requires the use of Reeks' computations and of experiments to develop a simple relation for the effect of $u_T/(\overline{u_r^2})^{1/2}$ on $(\overline{v_r^2})^{1/2}/(\overline{u_r^2})^{1/2}$. This approach for calculating k_D for dilute suspensions has been supported in recent experiments by Hay et al. (1996), Hay et al. (1998).

A surprising and, perhaps, the most interesting finding coming from an examination of measurements of k_D for annular flows is the influence of drop concentration. Early experiments by Namie and Ueda (1972) showed that k_D decreases with increasing concentration. These results were interpreted by arguing that drops dampened gas phase turbulence. Subsequent studies by Andreussi and Zanelli (1976), Govan et al. (1988) and Schadel et al. (1990) confirmed the findings of Namie and Ueda. In fact, these studies show that, for liquid volume fractions greater than about 3×10^{-4}, the deposition varies as C^{-1}; i.e., the rate of deposition is a constant (Schadel et al., 1990). An equation of the form

$$\frac{1}{k_D} = \left[\left(\frac{1}{k_{D0}}\right)^3 + \left(\frac{1}{k_{D\infty}}\right)^3\right]^{1/3} \tag{23}$$

represents the measurements where k_{D0} is the deposition parameter for $C \rightarrow 0$ and $k_{D\infty}$ is the deposition parameter for $C \rightarrow \infty$. In order to use this equation, a relation for $k_{D\infty}$ is needed.

One possible explanation for the behavior at large concentrations is that droplet coalescence results in the formation of large sluggish droplets which have very small $\overline{v_r^2}$. According to this argument, there is a critical drop concentration, beyond which further addition of liquid to the gas phase produces large drops but does not result in larger deposition rates. These notions prompted a study by Hay et al. (1996) in which dropsize distributions were measured over a large range of liquid flows for a fixed gas velocity, with equipment in which deposition rates had been measured. The droplet concentration profiles were flat for all conditions that were investigated, so the decrease in k_D could be associated with a decrease in $\overline{v_r^2}$, as indicated by

(19). The measured increases in dropsize with increasing liquid flow and with increasing distances along the tube did not show changes which would be consistent with the observed decrease of k_D (or of $\overline{v_r^2}$). Measured mean velocity profiles of the gas are similar to what would be found for flow over a roughened boundary; these suggest that the gas phase turbulence is not highly damped.

One possible explanation, put forward by Hay et al. (1996), is based on the fact that the motion of the droplets is quite sluggish; the mean square of their velocity fluctuations is the order of 1 per cent of the mean-square of the fluid velocity fluctuations. Since the number of encounters that a drop experiences per unit time, n, increases linearly with the number of drops per unit volume, N, it is plausible to explore the notion that particle–particle encounters result in a decrease in particle turbulence through inelastic interactions.

Turbulent time scales of the particles are large compared to the turbulent times scales of the fluid. Therefore, a long time is needed for drops to become fully entrained in the fluid turbulence. The small measured change in dropsize along the pipe suggests that a surprisingly small fraction of the drop interactions leads to coalescence (see Brown, 1985). Suppose that the encounter with another drop most often results in an interruption of the radial motion of the drop. This momentarily decouples the particle from the fluid velocity field. Fluid turbulence interacts with the particle and starts it on a new trajectory. Based on these arguments, Hay et al. developed the following relation for the limit of large N:

$$\left(\overline{v_r^2}\right)^{1/2}=\sqrt{\frac{2}{3}}n^{-1}\beta\left(\overline{u_r^2}\right)^{1/2} \tag{24}$$

Since $n \sim c$ the deposition constant varies as c^{-1}. Eq. (24) has not confirmed, nor has it been successfully developed into a usable predictive method. Clearly, this problem needs more attention.

3.5. Deposition in horizontal annular flow

A consideration of deposition in horizontal annular flows requires that the direct effect of gravitational settling on deposition be taken into account. Measurements of concentration profiles of drops (Williams et al., 1996; Paras and Karabelas, 1991) again show good mixing in that the concentration is roughly constant in planes perpendicular to the gravity vector. However, there is a variation of the concentration in the vertical direction, $C(y)$, which results in a variation of the concentration at the wall, $C_W(\theta)$.

An equation for V similar to (17) can be used. Again, the dispersed drops will be represented by a single size. An important difference is that the assumed Gaussian distribution will have a mean velocity of $|u_T|\cos\theta$, where $\theta = 0$ is the bottom of the pipe and $|u_T|$ is the absolute value of the settling velocity. At the bottom of pipe $P(v_r)$ is skewed to positive v_r (settling aids deposition); at the top it is skewed to negative v_r (settling opposes deposition). The integration of (26) around the pipe circumference shows that gravity increases R_D even if C_W/C_B does not vary. However, when gravity is important, a stratification of the drops occurs. This causes a variation of C_W with θ that provides an even greater enhancement of deposition. An equation of the form

$$V = \left(\overline{v_r^2}\right)^{1/2} f\left(\frac{u_T \cos\theta}{\left(\overline{v_r^2}\right)^{1/2}}\right) \tag{25}$$

where $u_T \cos\theta/(\overline{v_r^2})^{1/2}$ can be developed from (17). For $u_T \cos\theta/(\overline{v_r^2})^{1/2} \to 0$ function equals $(2\pi)^{-1/2}$. For a large enough argument, the function equals $u_T \cos\theta/(\overline{v_r^2})^{1/2}$. Since, for horizontal flows, the concentration of drops at the wall can be a function of θ the local deposition rate is

$$R_D = V(\theta)\frac{C_W}{C_B}C_B, \tag{26}$$

where $\overline{v_r^2}$ is given by (10).

3.6. Deposition of aerosols

Before closing this discussion, I would like to return to the case of aerosol deposition by turbulence in a fully developed field. As indicated earlier this would involve situations where $0.1\tau_p^+ < 10$. In contrast to annular flow, the rate of deposition is strongly influenced by nonhomogenieties in the turbulence that exist in the viscous wall layer. The theoretical model proposed by Friedlander and Johnstone (1957) pictures aerosol particles moving by turbulent diffusion from the bulk to a free-flight plane at y_{ff} in the viscous wall region, at which they become disengaged from the turbulence and embark on a free-flight to the wall. The location of this free-flight plane was given as $y_{ff} = v_{ff}\tau_p$. Friedlander and Johnstone selected v_{ff} as the root-mean square of the fluid velocity fluctuations at the edge of the viscous wall region. All particles starting a free-flight were assumed to reach the wall and the concentration at the wall was taken as zero. The theory agrees qualitatively with measurements under conditions where inertial impaction dominates (McCoy and Hanratty, 1977):

$$\frac{k_D}{u^*} = 3.25 \times 10^{-4}\tau_p^2 \tag{27}$$

This experimental result shows a remarkable increase of k_D with air velocity and with particle diameter, $k_D \sim u^{*5}$ and $k_D \sim d_p^4$. Davies (1966) argued that v_{ff} should be taken as equal to the turbulence intensity at $y = y_{ff}$, but predicted deposition rates that are too small.

Theoretical work on this problem has mainly involved modifications of the theory of Friedlander and Johnstone. Progress was limited because laboratory studies of rates of deposition and concentration distributions do not yield sufficient information about the deposition process. The availability of direct numerical simulations of turbulent fields allows experiments, on aerosol dispersion and deposition, which directly show free-flights.

Brooke and Hanratty (1994) used a DNS of turbulent flow in a channel and followed the paths of particles, with $\tau_p^+ = 3,5,10$, that were admitted into the field at $y^+ = 40$. These revealed a process which is similar to what Friedlander and Johnstone described, but quite different in the details. Particles are transported to the wall by turbophoresis and not by turbulent diffusion. Actually turbulent diffusion opposes transport to the wall. Particles can start free-flight to the wall from a range of y^+. The optimum location is $y^+ = 9$. At any given

y^+ only a small population of the particles, that have the largest velocities, start a free-flight. Not all particles in free-flight impact on the wall. Most of them get trapped in the viscous sublayer. The trapped particles usually do not diffuse to the wall. Most often they diffuse out of the viscous sublayer to a region far enough from the wall that they can start a new free-flight, which could cause them to impact on the wall. A synthesis of these findings into a new comprehensive theory is not yet available.

4. Effect of pipe inclination on the formation of slugs

Considerable progress has been made in analyzing slug flow by using the cell model explored by Dukler and Hubbard (1975), Nicholson et al. (1978), Fan et al. (1993a) and others. Detailed measurements of pressure profiles allowed Fan et al. (1993b) to develop models for single slugs. The model of Andritsos and Hanratty (1987b) can be used to describe the stratified flow between slugs. Critical unsolved problems are the prediction of the frequency of slugging and the distribution of slug lengths. A recent thesis by Woods (1998) examined mechanisms by which slugs are formed in order to provide a physical basis for developing equations for frequency. One part of this study involved an examination of the effect of small downward inclinations of 0 to 0.8°.

A comparison of the behaviors in horizontal and inclined pipes is best done at the same liquid height, h_L, and the same gas velocity. The force of gravity in a downward inclined pipe causes the liquid to move at a higher velocity. For large u_G the conditions for the initiation of slugging are the same as for horizontal pipes. Therefore, only the results for superfluid gas velocities less than 6 m/s will be discussed.

The expectation, when these experiments were initiated, was that results for small inclinations would be the same as what is found for a horizontal pipe operating at the same u_G and h_L, even though the liquid velocities are different. However, the findings did not support this notion. The h_L at transition changed with inclination, as did the frequency of slugging.

The effects of small inclinations on wave structure was striking. At 0.5° waves were completely dampened. The 9.0 and 18.0 cm waves (described in Section 3.3) that dominated the interface for air and water flowing in a horizontal pipe were not observed. For horizontal flows at low liquid rates, slugs could form at least 40 pipe diameters downstream of the entry. At low gas velocities the Froude number characterizing the liquid flow is less than unity. Slugs at the downstream end of the pipe could be traced upstream to small frequency waves. A closer examination showed that they represent backward moving depression waves. The picture that evolves from these experiments is that the depletion of the liquid by the slug gives rise to a depression wave which is reflected from the inlet as an expansion wave. The time interval between slugs then equals the time for the rarefaction and expansion waves to restore the level of the liquid to the point that a slug can be formed.

For the case of a pipe inclined at 0.5°, the Froude number of the liquid is greater than unity at the transition to slugging; depression waves cannot exist. The slugs that appear at the downstream end of the pipe can be associated with low frequency disturbances that appear upstream. These are identified with waves, having $\lambda = 6–8$ m, which propagate in the direction of the mean flow. The frequency of these waves is the same as the frequency of slugging. The waves are swells which increase in height as they propagate downstream. Since they have such a long wavelength the crests look like a flat surface to small wavelength waves. Eventually the long wavelength waves reach a height such that the gas velocity over the crests has a value that is needed for a Kelvin–Helmholtz instability (6.7 m/s for air–water). Capillary-gravity waves that appear at the crest rapidly evolve into a slug. These observations appear to agree with studies by Kordyban (1977a,b) in which waves were introduced into a stratified flow.

The long wavelength viscous stability analysis of a stratified flow, developed by Lin and Hanratty (1986), agrees with the observed conditions for the transition to slug flow in slightly inclined pipes. Furthermore the transition, described above, is more consistent with this analysis than is the transition observed for horizontal pipes.

However, at present, the role of inclination on the damping of waves is not understood. The waves are generated by a Jeffreys mechanism, whereby energy is supplied by pressure variations in phase with the wave slope. If this is larger than the viscous dissipation, waves can exist. The larger liquid velocity in inclined flows gives rise to a larger energy dissipation, in the liquid, than would exist in horizontal pipes. This could prevent the evolution of Jeffreys waves. Furthermore, an analysis of wave generation needs to recognize that, inclined flows have larger wave velocities because the liquid is moving faster. Energy transfer to waves is influenced by location of the critical layer, where the wave velocity equals the gas velocity (Miles, 1957). Therefore, energy transfer rates from the gas to the waves could be different in inclined flows than in horizontal pipes.

5. Use of stochastic methods to describe particle behavior in a nonhomogeneous field

Significant progress has been made in describing the behavior of a particle in a homogeneous isotropic turbulence and in describing diffusion and deposition of particles that originate from a source located in the center of a pipe. There is a need to extend these results to nonhomogeneous fields, particularly in considering aerosol deposition. Furthermore, the direct application of results from studies of homogeneous fields opens questions about the use of Eulerian methods to describe particle suspensions, such as exist in annular flows or in sediment transport. In both cases particles enter the field by a complicated process, that is not described by a diffusion mechanism. They move around the flow field under the influence of fluid turbulence and gravity and, eventually, deposit. These processes might be more accurately described by Lagrangian methods wherein the concentration field is pictured as resulting from a distribution of sources along the wall. The critical problem is the description of the average outcome of a large number of particle trajectories originating from a source.

These concerns prompted research, undertaken in our laboratory by Ilias Iliopoulos (1998), to describe the dispersion of solid or fluid particles from a point source in a nonhomogeneous field. The system considered was a direct numerical simulation of fully developed turbulent flow in a horizontal channel. The x-axis is in the direction of mean flow and the y-axis is perpendicular to the wall. The source was located at $y^+ = 40$ in order to emphasize the influence of the highly nonhomogeneous field that exists in the viscous wall layer. The first set of computational experiments involved tracking the paths of a large number of fluid particles. Paths of solids spheres were then calculated by solving an equation of motion that includes only the effects of drag and of gravity.

$$\frac{dV_i}{dt} = -\frac{3}{4}\frac{\rho_p}{\rho_L}\frac{C_D}{d_p}|\vec{V}-\vec{U}|(V_i-U_i) - g\left(1-\frac{\rho_F}{\rho_p}\right)\rho_p \tag{28}$$

where V_i and U_i are components of the velocities of the particle and the fluid. The position of the particle is then obtained with the equation

$$\frac{d\vec{x}_p}{dt} = V_i(x_0 t) \tag{29}$$

The solution of (28) to obtain the particle velocity along a possible trajectory requires the specification of the initial velocity of the particle and the fluid velocity seen by the particle as it moves around in the field. The DNS is used to give the fluid velocity, $U_i(\vec{x},t)$, at the location of the sphere at a given time.

The results of these computer experiments were used by Iliopoulos to examine the possibility of using a modified Langevin equation developed by Thomson (1984) and Durbin (1983) to represent the change of velocity of a fluid particle in a turbulent field. Suppose $\vec{U}$ is the velocity of a fluid particle at $\vec{x},t$ which had a velocity $\vec{U}_0$ at $\vec{x}_0,t=0$. Define $\vec{U}=\bar{U}+\vec{u}$ where, $\bar{U}(y)$ is the mean velocity and $\vec{u}$ is the fluctuating velocity with component u_i. The equation describing the changes in u_i along a trajectory is

$$d\left(\frac{u_i}{\sigma_i}\right) = -\frac{u_i}{\sigma_i\tau_i}dt + d\mu_i \tag{30}$$

Here $\sigma_i(y)$ is the root-mean-square of u_i in an Eulerian framework and $\tau_i(y)$ is a time scale. The first term on the right side of (30) is a deterministic restoring force and the second term is a random number which is uncorrelated in successive time intervals. For a homogeneous field, σ_i,τ_i are constant and τ_i is the Lagrangian time scale, τ_L. The Lagrangian correlation coefficient is found to equal $\exp(-t/\tau_i)$.

From an ensemble average of the Langevin equation

$$\langle d\mu_y\rangle = (d\sigma_y/dy)dt \tag{31}$$

This imposes drift velocities which oppose the tendency of fluid particles to move from regions of high turbulence to regions of low turbulence. It may be interpreted as resulting from a mean pressure gradient. Similarly, higher moments of $d\mu_y$ can also be obtained directly from the Langevin equation. These give

$$\left\langle d\mu_y^2\right\rangle = \left[\frac{2}{\tau_y} + D\left(\overline{u_y^3}/\sigma_y^2\right)\right]dt$$

$$\left\langle d\mu_y^3\right\rangle = \left[\frac{3}{\tau_y}\frac{\overline{u_y^3}}{\sigma_y^3} + D\left(\overline{u_y^4}/\sigma_y^3\right) - 3D\sigma_y\right]dt$$

$$\left\langle \mathrm{d}\mu_y^4 \right\rangle = \left[\frac{4}{\tau_y} \frac{\overline{u_y^4}}{\sigma_y^4} - \frac{12}{t_y} + D\left(\frac{\overline{u_y^5}}{\sigma_y^4} \right) - 4\frac{\overline{u_y^3}}{\sigma_y^3} D(\sigma_y) - 6D\left(\frac{\bar{u}_y^3}{\sigma_y^2} \right) \right] \mathrm{d}t \tag{32}$$

where $D(\ ) = \mathrm{d}(\ )/\mathrm{d}y$ and an ergodic hypothesis is made.

The solution of (32) was implemented by representing $\mathrm{d}\mu$ as the sum of two Gaussian distributions

$$\mathrm{d}\mu = pN(\mu_1, \sigma^2) + (1-p)N(\mu_2, \sigma^2). \tag{33}$$

Terms p, μ_1, μ_2 and σ^2 are selected so as to give the correct $\langle \mathrm{d}\mu \rangle, \langle \mathrm{d}\mu^2 \rangle, \langle \mathrm{d}\mu^3 \rangle$ and $\langle \mathrm{d}\mu^4 \rangle$. Because of the use of an ergodic hypothesis all statistical properties except τ_i can be obtained from Eulerian statistics.

Calculations at different times were made of profiles of the concentration of fluid particles in the x- and y-directions, profiles of $\overline{u_x^2}$ and $\overline{u_y^2}$ and of Lagrangian correlation coefficients. Approximate agreement with the computations with the DNS was obtained by simply taking τ_i as a constant. However, better agreement could be obtained by allowing τ_i to vary with y. This was done by assuming, in the central region of the channel, that τ_i is a constant which scales with the ratio of the height of the channel to friction velocity. At the wall τ_i is assumed to be equal to the Eulerian time scale and to scale with wall parameters.

Calculations were done for trajectories of solid particles with $\tau_\mathrm{p}^+ = 20$, $\rho_\mathrm{p}/\rho_\mathrm{F} = 2650$, $\beta\tau_\mathrm{LF} = 0.7$ and $u_\mathrm{T}/v^* = 0.46$. The particles were given the same initial velocities as were used in the computer experiments. They were removed from the field when they hit the wall. In solving (28) and (29) the modified Langevin equation was used to model fluid velocity fluctuations seen by the particle. Calculations were made of particle turbulence, correlation coefficients, deposition rates and concentration fields by using the $\tau_i(y)$ obtained from modeling the dispersion of fluid particles. Approximate agreement with the DNS experiments was realized. However, a very close agreement was obtained by simply reducing τ_i by 30 per cent. The reduction can be justified, on physical grounds, because of the expected influence of u_T/v^* on the correlation coefficient of the fluid velocity fluctuations seen by the particle.

The success of such a simple equation to describe fluid velocity fluctuations in a Lagrangian framework is quite surprising when one considers the complex structure of the turbulence in the viscous wall region. It encourages further exploration of this approach to model the dispersion of fluid and solid particles in nonhomogeneous turbulence.

6. Conclusions

6.1. Flow regimes

The assumption that the front of a slug is a sudden expansion provides a prediction of whether an intermittent flow will have slugs or plugs. A consideration of the stability of a slug provides a critical $h_\mathrm{L} = h_\mathrm{LO}$ below which slugs cannot be formed. This could be the chief criteria to determine the transition to slugging at large gas velocities and at high gas densities. For gas velocities larger than the Kelvin–Helmholtz critical condition slugs will form if

$h_L > h_{LO}$. The long wavelength viscous analysis correctly predicts the initiation of slugs at u_G less than the Kelvin–Helmholtz critical velocity if $h_L > h_{LO}$. The transition to annular flow needs to consider conditions at which droplets are generated and the ability of droplets to wet the wall completely. It is probably necessary to recognize that the transition occurs over a range of gas velocities, particularly for large diameter pipes.

6.2. Rates of deposition

For $(v_0/2\beta) < 1$ deposition does not depend on the velocity with which drops enter the field. The rate can then be related to the motion of the drop that is imparted by the fluid turbulence. For $\tau_p^+ > 20$ deposition occurs by a free-flight which starts outside the viscous wall layer. It is controlled by two resistances, diffusion to the vicinity of the wall and free-flight to the wall. For vertical annular flows, mixing is so great that the concentration is uniform, so free-flight controls deposition. Under these circumstances $k_D \sim (\overline{v_r^2})^{1/2}$.

For dilute concentrations, theory is available to calculate $(\overline{v_r^2})^{1/2}$ and the particle diffusivity. However, measurements of k_D suggest that $(\overline{v_r^2})^{1/2}$ decreases with increasing drop concentration. At a volume fraction greater than 3×10^{-4} the remarkable result is obtained that R_D is constant with $k_D \sim C^{-1}$. A theory for this behavior is presented which takes account of this behavior—however, this theory has not been tested.

For $v_0/2\beta \gg 1$ droplets departing from the wall take a undirectional path to an opposite wall. The deposition parameter using this limiting condition has been estimated as $k_D \cong v_0[1 - (\frac{2R\beta}{v_0})]$.

A consideration of horizontal annular flows needs to include the direct contributions of gravity. For a uniform concentration field the aiding of deposition by gravity at the bottom of the pipe is just balanced by the opposition at the top. However, because of gravity the drops in the core are stratified. Consequently, there is a variation of the droplet concentration around the circumference which gives rise to a net enhanced rate of deposition.

Aerosols are characterized by $\tau_p^+ < 20$ Free-flights start in the viscous wall region, so the understanding of the effects of nonhomogeneities are of first order importance. Aerosols are transported to the wall by turbophoresis, by which particles move from regions of large turbulence to regions of low turbulence. Particles can start free-flight from a number of planes, but $y^+ = 9$ appears to be optimum. Many particles in free-flight do not reach the wall but get trapped in the viscous sublayer. To deposit, the trapped particles usually need to diffuse far enough away from the wall so that they can be entrained in a fluid motion which is large enough to carry them to the wall.

6.3. Formation of slugs in inclined pipes

Waves observed at low gas velocities for stratified flows in a horizontal pipe are completely damped by inclining the pipe downward by an angle as small as 0.5°. This damping is associated with the larger liquid velocities observed in an inclined pipe—but the mechanism is not understood. Slugs evolved from very long wavelength waves, whose appearance is predicted by a viscous linear instability analysis. The long wavelength waves precipitate a

Kelvin–Helmholtz instability at their crests. These results could resolve the paradox as to why a linear viscous stability analysis predicts the appearance of slugs for air–water flows in horizontal pipes—where observations on the mechanism of formation seem to be quite different from what is suggested by theory.

6.4. Use of stochastic methods to describe particle behavior in a nonhomogeneous field

A critical theoretical problem in analyzing annular flows is that the description of the distribution of particles by classical Eulerian methods is inconsistent with the physics, in that these methods do not recognize that drops are continuously being entrained and deposited. Because of this interchange, the drops do not reach a stationary state and the mean drift downward might not equal the free-fall velocity.

Lagrangian methods which represent the droplet behavior as resulting from a distribution of sources seems to offer a sounder approach. A stochastic method is required to describe the average behavior of a single source. This, in turn, requires a model for the random velocity field seen by a particle. Recent calculations indicate that this modelling can be done by using a modified Langevin equation.

7. Closure

Advances in analyzing multiphase flows involves the judicious use of theories on wave generation, particle dynamics and interfacial phenomena. These analyses improve as basic knowledge improves. This is illustrated by considering the prediction of flow regimes and the rate of deposition.

The former requires a knowledge of the stability of a stratified flow, the stability of a slug, the initiation of atomization and the influence of gravity and turbulence on the dispersion of drops and bubbles. The rate deposition depends on dropsize, droplet turbulence, droplet/droplet interactions, the mechanism of atomization and on the turbulence dispersion of drops under the influence of gravity.

Acknowledgements

The author acknowledges a continuing association with Shell Corporation. He became interested in multiphase problems during a summer spent in their laboratory in Emeryville. They have supported and encouraged his work through the awarding of a Shell Professorship for five years, through grants-in-aid, and through technical discussions. The National Science Foundation has had a continuing interest in our fundamental studies of waves and of particle dynamics. The Design Institute on Multiphase Flows provided the impetus to couple basic research results with design needs. Most importantly, the author would like to acknowledge that the recent work described in this paper was supported by the Engineering Program of the Department of Energy. This Program recognized that basic research in multiphase flow, closely

related to practical needs, could have a long range impact and took steps to ensure that research along this line was continued in my laboratories.

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